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Market Neutral Liquidity Provision: Open Review

Basil Maire,[*†] Marcus Wunsch [‡]

Reviewers: Reviewer A, Reviewer B

Abstract. The final version of the paper "Market Neutral Liquidity Provision" can be found in Ledger Vol. 9 (2024) 73-88, DOI 10.5195/LEDGER.2024.389. There were two reviewers involved in the review process, neither of whom has requested to waive their anonymity at present, and are thus listed as Reviewers A and B. After initial review by Reviewers A and B, the submission was returned to the authors with feedback for revision (1A). The author resubmitted their work and responded to reviewer comments (1B). As Reviewer A had already recommended publication, the paper was only returned to Reviewer B, who subsequently also recommended the paper be accepted, thus ending the peer review process. Author responses have been bulleted for reader clarity.

1A. Review

Reviewer A

Does this paper represent a novel contribution to cryptocurrency or blockchain scholarship?

Yes, important contribution(s)

Please briefly explain why you think the paper makes or does not make a novel contribution.

Impermanent loss, which this paper address, is the major issue for automated market makers

Is the research framed within its scholarly context and does the paper cite appropriate prior works?

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Yes

Please assess the article's level of academic rigor.

Good (not excellent but a long way from poor)

Please assess the article's quality of presentation.

Good (not excellent but a long way from poor)

How does the quality of this paper compare to other papers in this field?

This is a good or average paper.

Please provide your free-form review for the author in this section.

This paper addresses the important topic of hedging impermanent loss and proposes a novel strategy to mitigate it using options. It is applied to Concentrated Liquidity Automated Market Makers, which is the dominant AMM in DeFi. The paper introduces the problem of liquidity provisions to AMMs and concisely presents its conclusions. The next step for research would be to explore whether the rewards for liquidity providers outweigh the costs of employing options.

Reviewer B

Does this paper represent a novel contribution to cryptocurrency or blockchain scholarship?

Yes, important contribution(s)

Please briefly explain why you think the paper makes or does not make a novel contribution.

The paper delivers a practical solution for the hedging of IL for liquidity providers.

Is the research framed within its scholarly context and does the paper cite appropriate prior works?

Important references are missing

Please assess the article's level of academic rigor.

Good (not excellent but a long way from poor)

Please assess the article's quality of presentation.

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Good (not excellent but a long way from poor)

How does the quality of this paper compare to other papers in this field?

The paper ranks highly but it may not be among the most authoritative references in the field.

Please provide your free-form review for the author in this section.

Please see my report for both the editor and authors.

[Editor's note: Reviewer's report follows presently.]

1. General

The authors consider the static replication of the impermanent loss (IL) at a fixed time horizon T of providing liquidity to an AMM pool. This is a very relevant problem of liquidity providers for trading pools at decentralized exchanges. The paper delivers a practical solution for the hedging of IL for liquidity providers.

The paper is written in relatively clear and concise style. I find that the paper could make a valuable contribution to the DeFi literature. However, the paper needs a revision before the final decision. I make specific remarks and suggestions for the revision.

2. Specific remarks

1. Abstract: "The hedge portfolio consisting of options and futures requires a significant capital outlay compared to the amount of liquidity provided, but earns fees and carry." Be specific what kind of fees and carry the hedge portfolio can make. From the table 4, the hedge portfolio is net long options so that the option's carry is negative because of options' theta.
2. Page 2, 1st paragraph: "we see a gap in the literature for a rigorous derivation of the hedging portfolio for the liquidity provider in concentrated pools, coupled with practical applicability." There is a recent paper addressing this topic which should be referenced: Lipton A., Lucic, V, and Sepp, A (2024) "Unified Approach for Hedging Impermanent Loss of Liquidity Provision", SSRN https://ssrn.com/abstract=4887298
3. Page 2, last paragraph before Section 2: "The hedge portfolio is net short, resulting in a positive premium earned by the hedge." For clarity: the hedge portfolio is net long options, generating a positive payoff (when the initial stake is delta-hedged with a future, the hedge portfolio is long both put and call legs) at the expiry time T financed by paid options premiums at the hedge inception time.
4. Eq (1): x and y are not defined.
5. Page 3, footnote: "At the time of writing, this type of AMM has been copied 45 times". The correct word is "forked" not "copied".

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6. Page 4, 2nd paragraph: “Fees in Uniswap v3 are kept outside the pool so that, together with l being invariant, the liquidity provision portfolio can be accurately valued without additional assumptions given a price P , independent of the trading activity – unlike in v2.” Clarification: the only difference is that the fees for v3 are accrued separately and the fees for v2 are accrued to the pool balances. Typically, the fees are separated from the IL calculations, so the IL for both v2 and v3 can be represented as one-to-one function of token price P.
7. Eq (9): “By the Carr-Madan Formula, any function f(x) that is twice continuously differ- entiable can be represented as ...” Clarification: the function V(P) in Eq (8) is not twice differentiable, so that strictly speaking the Carr-Madan formula does not apply. Yet, when formulated in terms of the generalized derivatives, this formula is valid (see Appendix 6.7 in Lipton-Lucic-Sepp (2024)).
8. Before lemma 4.1. “other than P being a positive continuous process, hence the derivation of our hedge is “model-free”.” Clarification: Carr-Madan decomposition formula can be applied for any process Pt as long as function f(P) is twice continuously differentiable. The assumption of the price path continuity is necessary for the representation of the log contract when someone uses it to price the variance swap. For valuation of IL payoff no assumptions of price path continuity is necessary.
9. Section 5. Some discussion is needed for the following aspects.

   (1) the liquidity ranges PU and PL must be in the set of tradable strikes. Note that the grid of ranges for LP is also discrete and this grid may quite dense for pools with high fee tiers. What kind of discretization error we can get when placing at strikes traded on Deribit (1% − 5%)?

   (2) The formula in the second line of Eq (17) is a sum of puts and calls within the liquidity range [PU , PU]. In practice, the best liquidity is available for near at-the-money (ATM) and out of ATM puts and calls. In the-money puts and calls have much higher spreads and lower liquidity. The bid-ask spread must be analyzed.

   (3) If I understand correctly the numbers on Figure (1): to stake the LP worth of about 0.76BTC we need to provide about 2.5 BTC worth of collateral. Thus, for capital allocation of 3.25BTC only 23% (= 0.76/3.25) of it earns LP fees. This is too capital-inefficient, I find.

   (4) It appears that the hedged LP has 3 sources of carry: LP fees, cash-and-carry from futures hedges, and options theta. Could you construct a greed or a heatmap to understand the profitability of the hedge LP? In the end, options initial premia is fixed, perp carry is floating and LP fees are floating. A x-axis grid of LP fees vs y-axis grid of perp or term carry could be illustrative.

   (5) For hedge portfolio in Table 4, could you plot P&L of options + LP on a grid of spot prices at the hedge expiry. I am curious about the discretization error on such a sparse grid of strikes.

1B. Author Response

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Reviewer B

- Thank you very much for the detailed report, we believe these comments have significantly contributed to improving the paper. Below we explain how we addressed the specific points raised.

1. Abstract: “The hedge portfolio consisting of options and futures requires a significant capital outlay compared to the amount of liquidity provided, but earns fees and carry.” Be specific what kind of fees and carry the hedge portfolio can make. From the table 4, the hedge portfolio is net long options so that the option’s carry is negative because of options’ theta.

- Thank you for your comment. We have adjusted the abstract accordingly and also used this comment as an opportunity to make this point clearer throughout the text.

2. Page 2, 1st paragraph: “we see a gap in the literature for a rigorous derivation of the hedging portfolio for the liquidity provider in concentrated pools, coupled with practical applicability.” There is a recent paper addressing this topic which should be referenced: Lipton A., Lucic, V, and Sepp, A (2024) “Unified Approach for Hedging Impermanent Loss of Liquidity Provision”, SSRN `https://ssrn.com/abstract=4887298`

- Thank you for this comment. We presented this manuscript at the CfC St Moritz conference on 12 January 2024; the recent paper was posted on SSRN on 2 August 2024. We reference this preprint accordingly.

3. Page 2, last paragraph before Section 2: “The hedge portfolio is net short, resulting in a positive premium earned by the hedge.” For clarity: the hedge portfolio is net long options, generating a positive payoff (when the initial stake is delta-hedged with a future, the hedge portfolio is long both put and call legs) at the expiry time T financed by paid options premiums at the hedge inception time.

- Thank you for this comment. We have adjusted the text throughout the document to avoid confusion between “long put” and “short in the underlying”.

4. Eq (1): x and y are not defined.

- We have added the definitions and an example.

5. Page 3, footnote: “At the time of writing, this type of AMM has been copied 45 times”. The correct word is “forked” not “copied”.

- This has been adjusted accordingly.

6. Page 4, 2nd paragraph: “Fees in Uniswap v3 are kept outside the pool so that, together with l being invariant, the liquidity provision portfolio can be accurately valued without

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additional assumptions given a price P, independent of the trading activity – unlike in v2." Clarification: the only difference is that the fees for v3 are accrued separately and the fees for v2 are accrued to the pool balances. Typically, the fees are separated from the IL calculations, so the IL for both v2 and v3 can be represented as one-to-one function of token price P.

- Thank you for this clarification. We have adjusted the text accordingly and added a footnote with an explanation and reference to Fukasawa-Maire-Wunsch (2024).

17. Eq (9): "By the Carr-Madan Formula, any function f(x) that is twice continuously differ entiable can be represented as ..." Clarification: the function V (P) in Eq (8) is not twice differentiable, so that strictly speaking the Carr-Madan formula does not apply. Yet, when formulated in terms of the generalized derivatives, this formula is valid (see Appendix 6.7 in Lipton-Lucic-Sepp (2024)).

- Thanks, we have amended the text to reflect the generalized sense in which the Carr-Madan Formula applies in our context.

8. Before lemma 4.1. "other than P being a positive continuous process, hence the derivation of our hedge is "model-free"." Clarification: Carr-Madan decomposition formula can be applied for any process Pt as long as function f(P) is twice continuously differentiable. The assumption of the price path continuity is necessary for the representation of the log contract when someone uses it to price the variance swap. For valuation of IL payoff no assumptions of price path continuity is necessary.

- Thank you for this clarification. We have removed the superfluous adjective "continuous" to reflect the generality in which the statement holds.

9. Section 5. Some discussion is needed for the following aspects.
(1) the liquidity ranges PU and PL must be in the set of tradable strikes. Note that the grid of ranges for LP is also discrete and this grid may quite dense for pools with high fee tiers. What kind of discretization error we can get when placing at strikes traded on Deribit (1% − 5%)?

- We added "Appendix A: Uniswap V3 Ticksize" and a footnote in Section 5 ("Hedging In Practice") which address this point.

(2) The formula in the second line of Eq (17) is a sum of puts and calls within the liquidity range [PU , PU]. In practice, the best liquidity is available for near at-the-money (ATM) and out of ATM puts and calls. In the-money puts and calls have much higher spreads and lower liquidity. The bid-ask spread must be analyzed.

- Thank you for this comment. In fact, the bid-ask spread is already included in our analysis, cf. Appendix. We have added a corresponding explanation.

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(3) If I understand correctly the numbers on Figure (1): to stake the LP worth of about 0.76BTC we need to provide about 2.5 BTC worth of collateral. Thus, for capital allocation of 3.25BTC only 23% (= 0.76/3.25) of it earns LP fees. This is too capital-inefficient, I find.

- We have added the section “Capital Efficiency”. We clarify that a large portion of the return is driven by carry, and explicitly compute the return on capital.

(4) It appears that the hedged LP has 3 sources of carry: LP fees, cash-and-carry from futures hedges, and options theta. Could you construct a greed or a heatmap to understand the profitability of the hedge LP? In the end, options initial premia is fixed, perp carry is floating and LP fees are floating. A x-axis grid of LP fees vs y-axis grid of perp or term carry could be illustrative.

- Thank you for this comment. We have added a heat map on p. 10 (Fig. 2) for this purpose.

(5) For hedge portfolio in Table 4, could you plot P&L of options + LP on a grid of spot prices at the hedge expiry. I am curious about the discretization error on such a sparse grid of strikes.

- Thank you for this comment. In fact, this plot was already shown in Fig. 2 on p.9 of the original submission. It is now contained in Fig. 3 on p.10 and referenced in Table 4.

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